$\frac{i}{n+1}$ Only for: p-p $(F_n(x), F^*(x))$
percentile
"smoothed"

"Max covered loss" refers to X

Binomial-Beta

$N \sim Bin(m, q)$

$Q \sim Beta\begin{pmatrix} a \\ b \\ \theta \end{pmatrix}$ x claims

$Q \sim Beta\begin{pmatrix} a^* = a + x \\ b^* = b + m - x \\ \theta \end{pmatrix}$

3 Points for Emp Bayes

1. non-par have ugly $\hat{v}$
2. semipar have $\hat{v} = \bar{X}$
3. semipar Pois have $a^2 = s^2 - v^2$

$\overline{n-1}$ $\overline{n}$

$g(x) = a^x$

$g'(x) = \ln a \cdot a^x$

$\underline{VaR_p(X)}$ rd ↑

3 Points for Bayes/Bühl Lines

1. Avg. the line
2. Linear Bühl
3. Bayes has realistic bounds

$a, b = nq + .5 \pm \sqrt{npq}$

rounds: ↑ ↓

Can't pick geo

w/ MoM

$\sigma^2 \gtrsim \bar{X}$

Var(MLE)

Asympt. ⇒ Info

$I(\theta) = -E[l''_{\theta}(\theta)]$

Standard ⇒ Delta Method

$Var(g(X)) = \Delta^2_{g(x)} Var(X)$

or

$\hat{\lambda} = g(\bar{x}) \rightarrow$ differentiate $\bar{X}^{-1}$ to $-\bar{X}^{-2}$

$Var((X-d)_+)$ of $X \sim exp(\theta)$

$(X-d)_+ = \begin{cases} 0 & x < d \\ X-d & x > d \end{cases}$

$Var((X-d)_+) = E[Var(Y|\theta)] + Var(E[Y|\theta])$

$= \theta^2 P(X > d) + (\theta - 0)^2 P(X < d) P(X > d)$

↑
(memoryless & conditioned on being > d)

Log-Likelihood

$2(l_F - l_0) \sim \chi^2_{df = \# new}$

"King of the hill, adding param"

Bühl = Bayes

For any conj priors

"Like χ^2" – no adj

"Smirnoffed Darling is needy" – needs adj

"Darling is a slut" → $n \to \infty$ irrelevant & heavy tail weight

$TVaR_p = e_L(\pi_p) + \pi_p$

$= \frac{E[L] - E[L \wedge \pi_p]}{1-p} + \pi_p$

$TVaR_p(aX - b) = a\,TVaR_p(X) - b$

$P(z) = E[z^x]$ $\quad P'(0) = p_1$

$P'(1) = E[X]$

Bootstrap

$MSE = E_{emp}(\hat{\theta}_i - \theta_{emp})$

varies ($\hat{\theta}_i$), constant (θ_{emp})

YP is already conditional

~~10/3/4~~ Formulas

- Kurtosis $\frac{E[(X-\mu)^4]}{\sigma^4}$ → expand it

- $E[X \wedge d]$ is the weirdone
$= \int_0^d x\,f(x)\,dx + d\,P(X>d)$
(or $= \int_0^d S(x)$ as all others do)

- Consistent if ① bias → 0
② Var → 0

- Empirical / Unspecified / Ogive / A-D $F_n(x)$ (split integrals) / $D(x) = F_n(x) - F^*(x)$ → $\frac{i}{n}$
- Smoothed / Percentile Match / p-p plot $(F_n(x), F^*(x))$ → $\frac{i}{n+1}$

- Fit MoM to dist
↳ no geo (other 3 ok)

Var (MLE)

$Var(\theta) = I^{-1}(\theta)$

$I(\theta) = -E[\ell''(\theta)]$

- Asymptotic: Info

2nd Derivs

$$I(\theta) = - \begin{pmatrix} E\left[\frac{d^2}{d\theta_1^2}\ell(\theta)\right] & E\left[\frac{d^2}{d\theta_1\theta_2}\ell(\theta)\right] \\ E\left[\frac{d^2}{d\theta_1\theta_2}\ell(\theta)\right] & E\left[\frac{d^2}{d\theta_2^2}\ell(\theta)\right] \end{pmatrix}$$

- Standard: Delta Method

1st Derivs

eval $[g'(x)]^2$ @ $E[X]$

$Var(g(x)) = [\Delta_\theta(x)]^2 Var(X)$

$\cancel{Var(g \quad) = \Delta_\theta^2 Var(\hat\theta) + \Delta_\alpha^2 Var(\hat\alpha) + 2\Delta_\theta\Delta_\alpha Cov(\alpha,\theta)}$

Binomial-Beta

read $f(x)$ CAREFULLY (denom x)

$N \sim Bin(m, q)$ w/ x claims
$q \sim Beta(a, b)$

n exposures ...

POSTERIOR → $q \sim Beta\left(\begin{matrix} a^* = a + x \\ b^* = b + m - x \end{matrix}\right)$

claims / non-claims

Normal-Normal

$X \sim Normal(\theta, v)$
$\theta \sim Normal(\mu, a)$

$v = E[Var(x)]$
$a = Var(E[x])$

Added Fall '0... IMPORTANT

For Ln-N, translate Ln to N

Pred $X \sim Normal(P_c, (1-Z)a + v)$ → works for 0 loss in 0 years too

~~Pr X ~ Normal $(\mu + Z(\bar{X}-\mu), (1-Z)a + v)$~~

Empirical Bayes 3 Step Summary

① All non-par use ugly v, μ
② All semi-par use $\hat{v} = \hat{\mu} = \bar{X}$
③ Semi-par uni has simple $\hat{a} = s^2 - \hat{v}$

important if denom a = 0

$n-1$ / n for Average

Chi-Square

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

EXCEPT if yr is a column → $\sum \frac{(O-E)^2}{V}$

yr | n | x

Minimize

$\ell(\theta) = MSE$ → mean
$\ell(\theta) = |\hat\theta - \theta|$ → median

Coherence - "Expect coherent tails"

E[X] Yes
TVaR Yes

- Splice

"Continuous" $\rightarrow f^-(x) = f(x)$

- $X \sim exp(\theta)$

$$\sum_1^n X \sim Gamma\left(\begin{matrix}\alpha = n \\ \theta\end{matrix}\right)$$

MSE & bias hats FIRST

- $E[(\hat{\theta}-\theta)^2|\theta]$ of $X \sim Pareto(\alpha=5, \theta)$, $\hat{\theta} = 6X^2$

$= E[(6X-\theta)^2|\theta]$

$= E[36X^4 - 12X^2\theta^2 + \theta^4|\theta]$

$= 36E[X^4] - 12E[X^2]\theta^2 + \theta^4$

- $g(x) = a^x$

$g'(x) = \ln a \cdot a^x$

- $Cov(X_1, X_2) = a$

- CI for $VaR_p(x)$

$a, b = nq + .5 \pm z\sqrt{nq(1-q)}$

round↓ round↑

- Do NOT apply v adj for ded to N, apply to PARAMETER: $N \sim NB(r, v\beta)$

MLE = MoM for:

"Poisson a NEGative ENGlish individual"

Poisson
Neg Bin
Exp
Normal
Gamma

(No grouped data)

- "χ^2 is more robust w/o adj"

Only χ^2 can handle groups, discrete, auto adj when fitting

- Inflation into scale parameter θ

- "Maximum Covered Loss" refers to ground-up NOT max PMT

- For $Var((X-d)_+)$ of $X \sim exp(\theta)$

Set $(X-d)_+ = \begin{cases} 0 & X<d \\ X-d & X>d \end{cases}$

$Var[(X-d)_+] = E[Var(Y|\theta)] + Var(E[Y|\theta])$

$\overset{exp}{=} \theta^2 P(X>d) + (\theta-0)^2 P(X<d)P(X>d)$

- $\int e^{xu} \cdot f(x)\,dx = M(u)$

MGF @ u

- Normal approx
  ↳ continuity correction
    ↳ to match ABCDE

- Log Likelihood Test

Reject l_0 for l_1 if $2(l_1 - l_0) > \chi^2_{df=\#new\ param}$

Go through models as "king of the hill" starting w/ LEAST param

(note: l_0 & l_1 will be neg – that's ok!)
↳ log-likelihood

*Want high (neg log-likelihood) ↓ high likelihood

"entries occur @ interval boundaries"

d_j @ beg
u_j @ end $\Rightarrow$ so d_0 @ 0, u_0 @ 1

- Never do quadratics with $a<0$ (get pos a)

- Profit problems: set up #years & little details CAREFULLY

- "Bühlshit": Bühl approxes Bayes

- The Properties For Measuring Shit
  - Translated Invariance $\rho(X+c) = \rho(X) + c$
  - Positive Homogeneity $\rho(cX) = c\rho(X)$
  - Monotonicity For $X \le Y$, then $\rho(X) \le \rho(Y)$ ← SD fails
  - Subadditivity $\rho(X+Y) \le \rho(X) + \rho(Y)$ ← VaR fails

$$\hat{\theta}_{MLE} \overset{exp}{=} \frac{\$}{\#\ \text{uncensored losses}}$$

(Truly \$, even with deds)

"Like χ^2"

Comparing Fit Tests

"Smirnoffel Darling" ds noety

K-S	A-D	χ^2	Likelihood Ratio
Ind	Ind	Ind & Group	Ind & Group
Censor : Crit ↓	Censor : Crit ↓	no change	no change
$n \to \infty \Rightarrow$ Crit ↓	no change	no change	no change
Fixed date : Crit ↓	Fixed date : Crit ↓	no change	no change
	More weight on tails	More weight on low prob group	

Darling is a ...
↓
$n \to \infty$ irrelevant & gen tail

- PGF $\quad P'(0) = p_1$

$P(z) = E[z^X]$

$P'(1) = E[X]$

$P''(1) = E[X(X-1)]$

$P'''(1) = E[X(X-1)(X-2)]$

⋮ ⋮

Bootstrap

- $MSE = E_{emp}\left[\left(\underset{\text{Varies}}{\hat{\theta}_i} - \underset{\text{Constant}}{\theta_{emp}}\right)^2\right]$
- Stupid estimators ONLY for $\hat{\theta}_i$, θ_{emp} is really the median/real value

- $TVaR = E[L \mid L > \pi_p]$

$\quad = E[L - \pi_p \mid L > \pi_p] + \pi_p$

$$\boxed{\begin{aligned} TVaR &= e_L(\pi_p) + \pi_p \\ &= \frac{E[L] - E[L \wedge \pi_p]}{1-p} + \pi_p \end{aligned}}$$

- $TVaR(aX - b) = a\,TVaR(X) - b$

Bühl = Bayes

For any conj prior situations

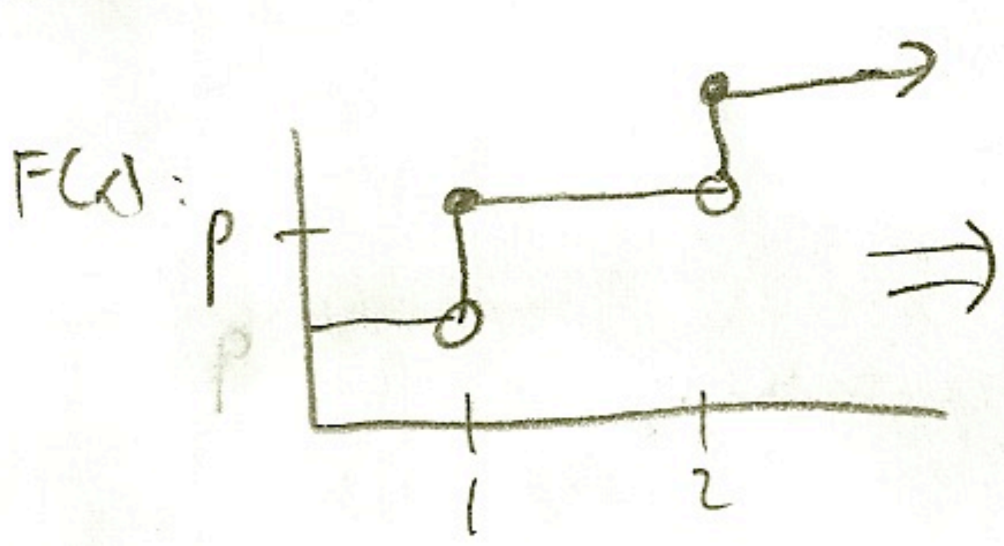
Claims experience & uni X

Bounds change to make sense!

(ASM V, 2)

Percentile

Anywhere $F(x)$ equals OR jumps over



$\Rightarrow \pi_p = [1, 2]$

closed

Not open

Bootstrap (special case)

MSE of Mean : $\dfrac{\sum (X - \bar{X})^2}{n^2}$

$MSE = E\left[\left(\hat{\theta}_i - \theta\right)^2 \mid \theta\right]$

Varies $\quad$ Constant $\quad$ plug in

1. Translational Invariance $\quad \rho(x+c) = \rho(x) + c$
2. Positive Homogeneity $\quad \rho(cx) = c \cdot \rho(x)$
3. Monotonicity $\quad X < Y \to \rho(X) < \rho(Y)$
4. Subadditivity $\quad \rho(X+Y) \le \rho(X) + \rho(Y)$ $\quad$ "Together is not more"
↓
No synergy