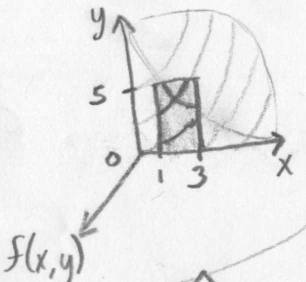
## Joint $f(x,y)$

$$P_r(1<X<3 \cap 0<Y<5) = \int_0^5\int_1^3 f(x,y)\,dx\,dy$$




$$E[XY] = \int_y\int_x x\cdot y\cdot f(x,y)\,dx\,dy$$

### Covariance

$$Cov[X,Y] = E[XY] - E[X]\cdot E[Y]$$

(only for indep $+ E_Y$)

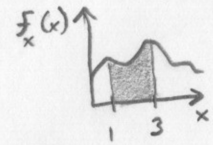
(Used in $Var[aX+bY+c] = a^2 Var[X] + b^2 Var[Y] + 2ab\,Cov[X,Y]$)

## Marginal $f_X(x)$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

opposite variable

$$P_r(1<X<3) = \int_1^3 f_X(x)\,dx$$




$$\left.\begin{aligned} E[X] &= \int_{-\infty}^{\infty} x\cdot f_X(x)\,dx \\ E[X^2] &= \int_{-\infty}^{\infty} x^2\cdot f_X(x)\,dx \end{aligned}\right\} \begin{aligned} &Var[X] \\ &= E[X^2] - E[X]^2 \end{aligned}$$

$$f(x,y) = f_X(x)\cdot f_Y(y) \overset{\text{iff}}{\Longleftrightarrow} \text{indep}$$

## Conditional $f_{X|Y}(x|Y=y)$

From $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

$$f_{X|Y}(x|Y=y) = \frac{f(x,y)}{f_Y(y)}$$

$\Rightarrow$ Remember, you can rearrange with algebra!

From joint,
To find $Var[X|Y=y]$, get to marginal pdF **FIRST** to get to conditional

$$= E[X^2|Y=y] - E[X|Y=y]^2$$

$$= \int_{-\infty}^{\infty} x^2\cdot f_{X|Y}(x|Y=y)\,dx - \left[\int_{-\infty}^{\infty} x\cdot f_{X|Y}(x|Y=y)\,dx\right]^2$$

## Moment-Generating Function $M_X(t)$

Technique A: $M_X(t) = E[e^{tX}] = \int_{-\infty}^{\infty} e^{tx}\cdot f_X(x)\,dx$

Technique B: $M_X(0) = 1$

$$\left.\begin{aligned} M'(0) &= E[X] \\ M''(0) &= E[X^2] \end{aligned}\right\}$$

$$M^{(n)}(0) = E[X^n]$$

Mixed Distribution : half continuous, half discrete piecewise

Mixture of Distributions : Weighted average of distributions
(weights $\alpha_1, \alpha_2, \dots$)

Use $\alpha$ for: $E[X]$ $f(x)$ $F(x)$ $M_x(t)$

NEVER SQUARE $\alpha$
NEVER $\alpha$ Var

But NOT FOR Var(x)

$$Var(X) = E[X^2] - E[X]^2$$

$$\left[\alpha_1 E[X_1^2] + \alpha_2 E[X_2^2]\right] \qquad \left[\alpha_1 E[X_1] + \alpha_2 E[X_2]\right]$$

Sums of RV's

$E[aX+bY+c] = aE[X] + bE[Y] + c$

$Var[aX+bY+c] = a^2 Var[X] + b^2 Var[Y] + 2ab\, \underbrace{Cov[X,Y]}$

(0 iff indep)

+, not -, signs from a & b carry!

(like FOIL)

$Cov[aX+bY+c, dZ+eW+f] = ad\,Cov[X,Z] + ae\,Cov[X,W] + bd\,Cov[Y,Z] + be\,Cov[Y,W]$

May use Convolution Method ("circuits" & "back-ups")

$$f_Y(y) = \int_{-\infty}^{\infty} f_1(x_1) \cdot f_2(y - x_1)\, dx_1$$

$Y = X_1 + X_2$

with pdf $f_1(x)$    with pdf $f_2(x)$

$X_2 = Y - X_1$

(Use $X_2$ pdf)

($dx_1$ bc we rewrote $X_2$ in terms of $X_1$)

For $Y = X_1 + X_2 + \dots + X_n$

$$Var[Y] = \sum_{i=1}^{n} Var[X_i]$$

$$M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t) = M_{X_1}(t) \cdot M_{X_2}(t) \cdot \dots \cdot M_{X_n}(t)$$

# Misc. Formulas

$$Cov[X,Y] = \sigma_{X,Y} = E[XY] - E[X]\cdot E[Y]$$

**Correlation Coeff**

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \cdot \sigma_Y}$$

- $\rho_{X,Y} = 0 \Leftrightarrow$ indep
- $-1 \le \rho_{X,Y} \le 1$

**Coefficient of Variation**

$$= \frac{\sigma_x}{\mu_x}$$

$\rightarrow$ (Std. dev. not variance in numerator)

**Chebyshev's Inequality**

$$P\left(|X-\mu| \ge r\sigma\right) \le \frac{1}{r^2}$$

**Transformations**

$$f_Y = f_X(v(y)) \cdot |v'(y)|$$

**Min/Max**

$U = \max(X,Y)$

$$\begin{aligned} F_U(u) &= P(\max(X,Y) \le u) \\ &= P\left[(X \le u) \cap (Y \le u)\right] \\ &= F_X(u) \cdot F_Y(u) \end{aligned}$$

"Max CDF"

$V = \min(X,Y)$

$$\begin{aligned} F_V(v) &= P(\min(X,Y) < v) \\ &= 1 - P(\min(X,Y) > v) \\ &= 1 - P(X > v) \cdot P(Y > v) \\ &= 1 - [1 - F_X(v)] \cdot [1 - F_Y(v)] \end{aligned}$$

"Min Survivor"