$$\ddot{a}_x^{(m)} = \ddot{a}_x - \frac{m-1}{2m}$$

$$\cancel{P_x = P^1_{x:\overline{n}|} + P_{x:\overline{n}|}^{\ \ 1}\,{}_nV_x}$$

Percentiles → Normal? → ${}_tp_x$ match?

$${}_tV_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}$$

$$Var(S) = E(N)\,Var(X) + Var(N)\,E(X)^2$$

SQUARED, ~~NOT~~ NOT SECOND MOMENT

Markov Var

npq

dM $\ddot{a}_x \Rightarrow$ ① Get $A_x = \frac{a_{\overline{\omega-x}|}}{\omega-x}$

② Then $\ddot{a}_x = \frac{1-A_x}{d}$

* Get endow A for $\ddot{a}_{x:\overline{n}|}$

$$\mathring{e}_{30:\overline{50}|} = \int_0^{50} {}_tp_x\,dt$$

$$= \frac{\int_{30}^{80} s(x)\,dx}{s(30)}$$

Conditional

$$M \begin{cases} \sim & 0,5 \\ \sim & 5,15 \end{cases}$$

$$e = \int_0^5 \sim + {}_5p_x \int_0^{10}$$

"deferred memoryless"

Poisson always indep

(*) Don't treat dM PTG as memoryless

"Joe Pa defers"

UDD M

$$\frac{q_x^{(j)}}{\ln(p_x'^{(j)})} = \frac{q_x^{(\tau)}}{\ln(p_x^{(\tau)})}$$

$$Cov(T_{xy}, T_{\overline{xy}}) = (\mathring{e}_x - \mathring{e}_{xy})(\mathring{e}_y - \mathring{e}_{xy})$$

$$\mathring{e}_x = \mathring{e}_{x:\overline{5}|} + {}_5p_x\,\mathring{e}_{x+5}$$

Reversionary Annuity

Pay 40 after 30 dies $= \bar{a}_{40} - \bar{a}_{40:30}$

$${}^2\ddot{a}_x = \frac{1 - {}^2A_x}{{}^2d}$$

$$T_x = \int_0^\infty l_{x+t}$$

$$\mathring{e}_x = \frac{T_x}{l_x}$$

UDD S

$${}_tp_x'^{(2)} = (1 - t\cdot q_x'^{(2)})$$

$$\mathring{e}_x \overset{dM}{=} \frac{\omega - x}{\alpha+1}$$

$$\ddot{a}_x^{(m)} = \alpha(m)\,\ddot{a}_x - \beta(m)$$

Bernoulli

$$Var(X) = pq(X_p - X_q)^2$$

For ${}_nq_{\overline{xy}}$ or ${}_{n|}q_{\overline{xy}}$

use p's → $p_{\overline{xy}} = p_x + p_y - p_{xy}$

For G, look for $\ddot{a}_x$ vs. $\ddot{a}_{x:\overline{n}|}$

Math

$$\sum_{t=0}^{n} a^t = \frac{1-a^{n+1}}{1-a}$$

Triple Premium

WL, Term, EI, Paid up

PE, WL

Accum Prem = Reserve @n + old pmts

① $P_x\,\ddot{s}_{x:\overline{n}|} = {}_nV_x + {}_nk_x$

② $P_{x:\overline{n}|}\,\ddot{s}_{x:\overline{n}|} = {}_nV_x + {}_nk_x \quad (= 1 + {}_nk_x)$

③ $P^1_{x:\overline{n}|}\,\ddot{s}_{x:\overline{n}|} = {}_nV^1_{x:\overline{n}|} + {}_nk_x \quad (= 0 + {}_nk_x)$

④ ${}_nP_x\,\ddot{s}_{x:\overline{n}|} = {}^n_nV_x + {}_nk_x \quad (= A_{x+n} + {}_nk_x)$

* Then difference these eqtn's

Poisson Gamma

T is time til event

$E[T] = \alpha\theta$

$E[T^2] = \alpha(\alpha+1)\theta^2$

$Var[T] = \alpha\theta^2$

- α is # of events
- $\alpha = 1 \sim \exp$
- $\theta = \frac{1}{\lambda}$

$$\boxed{f(x) = \frac{x^{\alpha-1} e^{-\frac{x}{\theta}}}{\theta^\alpha\,\Gamma(\alpha)}}$$

$\Gamma(\alpha) = (\alpha-1)!$

(1) for any $_{n}q_{\overline{xy}}$ or $_{n|}q_{\overline{xy}}$

use p's $\Rightarrow$ $p_{\overline{xy}} = p_x + p_y - p_{xy}$

(6) ~~Match premium for each~~ wval

$\pi = b_t \cdot v q_x$ will wash out on prem & APV (benefits)

(7) (68) $\boxed{\ddot{a}_x^{(m)} = \ddot{a}_x - \frac{m-1}{2m}}$

(13) percentiles can be $_{t}p_x = .75$ -or- Normal Approx.

Find t

(16) $\boxed{_{t}V_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}}$

very useful for ill

(need level prem/ben from x to $x+t$)

(27) L is the prospective loss, forget the past!

(31) $\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$ (When asked for $e_{\overline{xy}}$ use this)

(43) $\boxed{A_x \overset{dM}{=} \frac{\ddot{a}_{\overline{\omega-x}|}}{\omega-x}}$

$\boxed{\ddot{a}_x = \frac{1-A_x}{d}}$

Get a^{dM} from $\boxed{A_x \overset{dM}{=} \frac{\ddot{a}_{\overline{\omega-x}|}}{\omega-x}}$

$\boxed{_{t}p_x \overset{dM}{=} \frac{\omega-x-t}{\omega-x}}$

$\boxed{\mu_{x+t} \overset{dM}{=} \frac{1}{\omega-x}}$

(46) Careful of $\ddot{a}$ vs. a

(56) $\boxed{(\bar{I}\bar{A})_x = \int t \cdot e^{-\delta t} {}_{t}p_x \, \mu_{x+t} \, dt}$

(71) For linearly changing π, use avg. π

$\boxed{A_x \overset{CM}{=} \frac{\mu}{\mu+\delta}}$

$\boxed{\ddot{a}_x \overset{CM}{=} \frac{1}{\mu+\delta}}$

(72) For $_{10|}^{2}A_x$, just look @ $_{10|}A_x$ & double all δ

(77) $\boxed{P_x = P^1_{x:\overline{n}|} + P_{x:\overline{n}|}^{\ \ 1} \cdot {}_{n}V_x}$

endowment prems fund endowment

$\cancel{Var(S) = E[N]\,Var(X) + Var[N] \cdot (E(X))^2}$

(86) $a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - 1 + {}_{n}E_x$

(timeline it)

(88) $\boxed{\mathring{e}_x = p_x(1 + \mathring{e}_{x+1})}$

(104) $Var({}_{15}E_x) = v^{30} {}_{15}p_x \cdot {}_{15}q_x$

(116) (210) For $\mu \sim unif(0,n)$

$\int_0^n \int_0^t \mu e^{-\mu t} \, dt \, \frac{1}{2} \, d\mu$

(124) Poisson are independent, usually even when it looks like it's not

(131) $\mathring{e}_{x:\overline{n}|} = \int_0^n {}_{t}p_x \, dt$

(151) $\boxed{Var(Markov) = npq}$

(153) For n-yr endowment

$Var({}_0L) = Var(A_1) + Var(A_2) + 0$

(159) $\boxed{{}_{1}AS = \frac{(G - exp)(1+i) - 1{,}000\, q_x}{p_x}}$

(163) $\boxed{\sum_{t=0}^{n} (a)^t = \frac{1-a^{n+1}}{1-a}}$

(188) $\boxed{\mu_x = \frac{\alpha}{\omega - x}}$ for dM

(198) Expenses: $\boxed{Loss = v^T - G\ddot{a}_x + expenses}$

(207) $\boxed{\mathring{e}_{30:\overline{50}|} = \frac{\int_{30}^{80} s(x)\,dx}{s(30)}}$

(209) Variances can be additive, just avoid α Var

(241) For G, look for different terms esp. with h-pay

(243) $\boxed{ExpReserve = APV(exp - exp\ prem)}$

$\boxed{G = P + exp\ prem}$

(261) $\boxed{\bar{A}_{xy}^{\ 2} = \int v^t \cdot {}_{t}q_x \cdot {}_{t}p_y \, \mu_{y+t}}$

$x\overset{1}{y} : p$

$x\overset{2}{y} : q$

(272) $Cov = (\mathring{e}_x - \mathring{e}_{xy})(\mathring{e}_y - \mathring{e}_{xy})$

45 $\ddot{a}_x$ dM → get it from $A_x = \frac{a_{\overline{\omega-x}|}}{\omega-x}$

77 $P_x = P^1_{x:\overline{n}|} + P_{x:\overline{n}|}^{\;\;1} \cdot {}_nV_x$

81 $\boxed{Var(S) = E[N]\,Var(X) + Var(N)\,(E[X])^2}$ → Can be Applied to 101

$E[N] \to \lambda$, $Var(N) \to \lambda$

124 Poisson is VERY INDEP

139 Percentile can be normal OR ${}_tp_x$

187 $\dfrac{q_x^{(j)}}{\ln(p_x^{(j)})} \overset{UDD^M}{=} \dfrac{q_x^{(\tau)}}{\ln(p_x^{(\tau)})}$

234 UDD^S ${}_tp_x^{(2)} = (1 - t q_x^{(2)})$

150 $\mathring{e}_{\overline{xy}}$ → rewrite as $\mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$

151 Markov Var → npq

167 $\mu = \begin{cases} \sim^{0,5} \\ \sim_{5,10} \end{cases}$: $\int_0^5 \sim + {}_5p_x \int_0^5$

Memoryless deferred

207 $\mathring{e}_{30:\overline{50}|} = \dfrac{\int_{30}^{80} s(x)\,dx}{s(30)}$

$\mathring{e}_{30:50} \to \mathring{e}_{0:\overline{30}|} + {}_{30}p_0 \cdot \mathring{e}_{30:\overline{50}|} = \mathring{e}_{0:\overline{80}|}$

$\mu_{x+t}^{(\tau)} = \sum_{1}^{m} \mu_{x+t}^{(j)}$ Consider $(\mu'^{(j)} = \mu^{(j)})$

$\boxed{{}_tp_x^{(\tau)} = \prod_{1}^{m} {}_tp_x'^{(j)}}$

Prob Rate

$\cancel{{}_nq_x^{(j)} = \int_0^n {}_tp_x^{(\tau)} \mu_{x+t}^{(j)}}$

$\boxed{{}_tq_x^{(\tau)} = \sum_{1}^{m} {}_tq_x^{(j)}}$

$\cancel{q_x^{(j)} = q_x^{(\tau)}}$

$\boxed{f_{J|T}(j|t) = \frac{\mu_{x+t}^{(j)}}{\mu_{x+t}^{(\tau)}}}$

$f_{T,J}(t,j) = {}_tp_x^{(\tau)} \mu_{x+t}^{(j)}$

survive ALL of them

$\boxed{{}_nq_x^{(j)} = \int_0^n {}_tp_x^{(\tau)} \mu_{x+t}^{(j)}}$

$\boxed{\frac{q_x^{(j)}}{\ln(p_x'^{(j)})} \overset{UDD^M}{=} \frac{q_x^{(\tau)}}{\ln(p_x^{(\tau)})}}$

"Product P Prime"

- Primes: $\prod {}_tp_x'^{(j)} = {}_tp_x^{(\tau)}$
- No prime: $\sum {}_tq_x^{(j)} = {}_tq_x^{(\tau)}$

From prob to rates

1. ${}_tp_x^{(\tau)}$ from ${}_tq_x^{(j)} \longrightarrow \boxed{{}_tp_x^{(\tau)} = 1 - \sum_{1}^{m} {}_tq_x^{(j)}}$
2. $\mu_x^{(j)}$ from $\boxed{\frac{\frac{d}{dt}\left[{}_tq_x^{(j)}\right]}{{}_tp_x^{(\tau)}} = \mu_{x+t}}$
3. ${}_tp'^{(j)}$ From $\boxed{{}_tp_x'^{(j)} = e^{-\int_0^t \mu_{x+t}^{(j)}}}$
4. Check by ${}_tp_x^{(\tau)} = \prod {}_tp_x'^{(j)}$

From rates to probs

1. $\mu_x^{(j)}$'s from $\boxed{\mu_x^{(j)}(t) = \frac{-\frac{d}{dt}\left({}_tp_x'^{(j)}\right)}{{}_tp_x'^{(j)}}}$
2. ${}_tp_x^{(\tau)}$ from $\prod {}_tp_x'^{(j)}$ or $\boxed{e^{-\sum \mu^{(j)}}}$
3. $\boxed{{}_tq_x^{(j)} = \int_0^t {}_sp_x^{(\tau)} \mu_{x+s}^{(j)}\, ds}$

$${}_tq_x^{(j)} \longrightarrow \mu_x^{(j)} \longrightarrow {}_tp_x'^{(j)} \longrightarrow {}_tp_x^{(\tau)}$$

Integration by Parts

Formulas

4

$$\mu_x = \frac{f_x(x)}{s_x(x)} = -\frac{d}{dx}[\ln s_x(x)] \quad \longleftrightarrow \quad f_x(x) = s_x(x)\cdot\mu_x$$

$$f_{T(x)}(t) = {}_tp_x \cdot \mu_x(t)$$

$$s_x(x) = \exp\left(-\int_0^x \mu_s\, ds\right)$$
$${}_tp_x = \exp\left(-\int_t^{x+t} \mu_s\, ds\right)$$
$${}_tp_x = \exp\left(-\int_0^x \mu_x(s)\, ds\right)$$

$${}_{a|b-a}q_x = \int_a^b {}_tp_x\, \mu_{x+t}\, dt$$

(instead, do $\;{}_ap_x - {}_bp_x = e^{-\int_0^a \mu} - e^{-\int_0^b \mu}$)

Prob of Survival TO upper CONDITIONAL on FROM lower

For / Then: $\boxed{\mu_x'(t) = \mu_x(t) + k \quad {}_tp_x' = {}_tp_x \cdot e^{k}}$ & $\boxed{\mu_x'(t) = \mu_x(t)\cdot k \quad {}_tp_x' = ({}_tp_x)^k}$

6

$$\mathring{e}_{x:\overline{n|}} = \int_0^n {}_tp_x\, dt$$

$$E[(T(x)\wedge n)^2] = 2\int_0^n t\,{}_tp_x\, dt$$

7 Recursive

$$\mathring{e}_x = \mathring{e}_{x:\overline{n|}} + {}_np_x\, \mathring{e}_{x+n}$$

(A) Common Recursive

$$e_{x:\overline{n|}} = p_x + p_x \cdot e_{x+1:\overline{n-1|}}$$

$$\mathring{e}_{x:\overline{n|}} = e_{x:\overline{m|}} + {}_mp_x \cdot e_{x+m:\overline{n-m|}}$$

Life Table Concepts:

$$T_x = \int_0^\infty l_{x+t}\, dt \qquad {}_nL_x = \int_0^n l_{x+t} \qquad \mathring{e}_x = \frac{T_x}{l_x} \qquad e_{x:\overline{n|}} = \frac{{}_nL_x}{l_x}$$

Central Death Rate Concepts:

$$m_x = \frac{q_x}{1-.5q_x} \qquad {}_nm_x = \frac{{}_nd_x}{{}_nL_x}$$

$${}_nm_x \overset{CM}{=} \mu_x$$

8

UDD

$${}_sq_{x+t} \overset{UDD}{=} \frac{s\cdot q_x}{1-t\cdot q_x}$$

$${}_sp_x\, \mu_{x+s} \overset{UDD}{=} q_x$$

$${}_sp_x \overset{UDD}{=} e^{-s\mu_x}$$

$$l_{x+s} \overset{UDD}{=} l_x - s\,d_x$$

$$\mu_{x+s} \overset{UDD}{=} \frac{q_x}{1-s\,q_x}$$

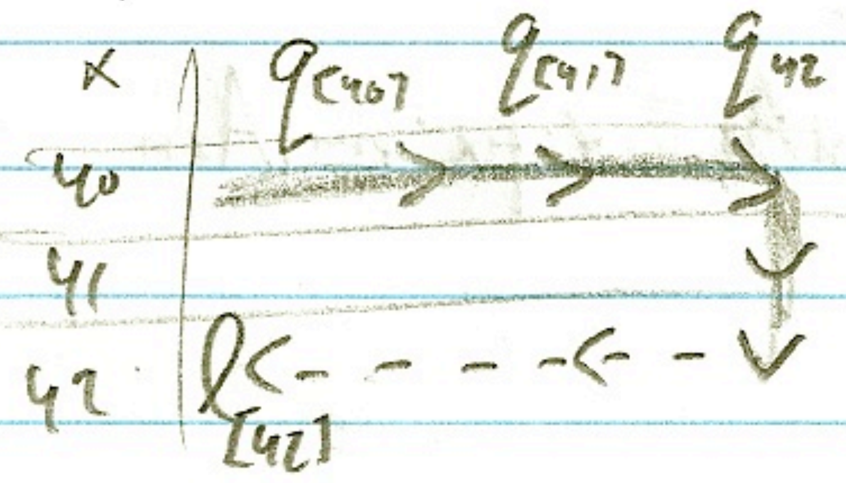
Phil says just be able to think about linear & that's enough

CM

$$p_x \overset{CM}{=} e^{-\mu}$$

Memory(u)

$${}_sp_{x+t} \overset{CM}{=} p_x^{\,s}$$



$Z_x = v^t$

[10] $\boxed{\bar{A}_x = \int_0^\infty v^t \cdot f_T(t)\, dt}$ ${}^2\bar{A}_x$: double δ

not necessarily 2nd moment (important in annuities)

• Know WL & P.E., derive others

$Var(\bar{A}_x) = {}^2\bar{A}_x - \bar{A}_x^2$

$\boxed{E[Z_x^n] = \int_0^\infty b_t^n \cdot v_t^n \cdot {}_tp_x \mu_{x+t}\, dt}$ $\boxed{\bar{A}_x \overset{CM}{=} \frac{\mu}{\mu+\delta}}$

---

[11] $Var(Z_3)$ Endow. Ins.

$Z_3 = \underset{\text{term}}{Z_1} + \underset{PE}{Z_2}$

$\boxed{Var(Z_3) = Var(Z_1) + Var(Z_2) + 2\,Cov(Z_1, Z_2)}$

Even outside normal,

$E[Z^n] = \int b^{tn} \cdot v^{tn} \cdot f_T(t)$

Apply moment to b AND v!

Normal Approx

• Scaling var:

• $x covered on 1 ins = $x^2 \cdot var$

• $1 covered on n ins = $n \cdot var$

Square dollars NOT PEOPLE

---

[13] $E[Z] = \sum_{k=0}^\infty {}_{k|}q_x \cdot v^{k+1} = \sum_{k=0}^\infty {}_kp_x \cdot q_{x+k} \cdot v^{k+1}$

$E[Z^2] = \sum_{k=0}^\infty {}_{k|}q_x \cdot v^{2(k+1)} = \sum {}_kp_x \cdot q_{x+k} \cdot v^{2(k+1)}$

Relate to WL

Def.) $\boxed{{}_{n|}A_x = v^n {}_np_x \cdot A_{x+n}}$

End Ins.) $\boxed{A_{x:\overline{n|}} = A_x - v^n {}_np_x A_{x+n} + v^n \cdot {}_np_x}$

---

[14] WL) $\boxed{A_x = v \cdot q_x + v \cdot p_x \cdot A_{x+1}}$ Recursive

End. Ins.) $A_{x:\overline{n|}} = v \cdot q_x + v \cdot p_x \cdot A_{x+1:\overline{n-1|}}$

Term) $A^1_{x:\overline{n|}} = v \cdot q_x + v \cdot p_x \cdot A^1_{x+1:\overline{n-1|}}$

Def) ${}_{n|}A_x = v \cdot p_x \cdot {}_{n-1|}A_{x+1}$

GR This

$(IA)^1_{x:\overline{n|}} = \sum_{k=1}^n k \cdot {}_{k-1|}A^1_{x:\overline{1|}}$

$(IA)^1_{x:\overline{n|}} = A^1_{x:\overline{n|}} + v \cdot p_x \cdot (IA)^1_{x+1:\overline{n-1|}}$

$(DA)^1_{x:\overline{n|}} = n \cdot A^1_{x:\overline{n|}} + v \cdot p_x \cdot (DA)^1_{x+1:\overline{n-1|}}$

$(DA)^1_{x:\overline{n|}} = A^1_{x:\overline{n|}} + (DA)^1_{x:\overline{n-1|}}$

accounts for pmt of 1 in last year

$$\boxed{\bar{A}_{x:\overline{n}|} = \frac{i}{\delta} A^1_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1}}$$ ← Trap: There is no $\bar{A}_{x:\overline{n}|}^{\ \ 1}$!

$$\boxed{{}^2\bar{A}_x = \frac{{}^2 i}{{}^2\delta} \cdot {}^2A_x \quad \left(\text{where } {}^2 i = i @ 2\delta, \quad {}^2\delta = 2\delta\right)}$$

16

WL) $\boxed{\bar{a}_x = \frac{1-\bar{A}_x}{\delta}}$ ⟷ $\boxed{\bar{A}_x = 1 - \delta \cdot \bar{a}_x}$ CPM: $\boxed{\bar{a}_x = \int_0^\infty v^t \cdot {}_t p_x}$

Term) $\bar{a}_{x:\overline{n}|} = \frac{1-\bar{A}_{x:\overline{n}|}}{\delta}$

Def) ${}_{n|}\bar{a}_x = \bar{a}_x - \bar{a}_{x:\overline{n}|}$

$$\boxed{{}_{n|}\bar{a}_x = \frac{\bar{A}_{x:\overline{n}|} - \bar{A}_x}{\delta}}$$

17 $\boxed{\ddot{a}_x = \frac{1-A_x}{d}}$ ⟷ $\boxed{A_x = 1 - d\ddot{a}_x}$ CPM: $\boxed{\ddot{a}_{x:\overline{n}|} = \sum_{k=1}^{n} v^{k-1} \cdot {}_{k-1}p_x}$

Generally: $\boxed{E[Y] = \sum_{t=1}^{\infty} b_t \cdot v^t \cdot {}_{t-1}p_x}$

$$\boxed{\ddot{s}_{x:\overline{n}|} \cdot {}_nE_x = \ddot{a}_{x:\overline{n}|}}$$

(v-like PV factor)

Imm.

$$\boxed{a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n+1}|} - 1}$$

$$\boxed{a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x}$$

$$\boxed{s_{x:\overline{n}|} = {}_1E_{x+n} \cdot s_{x:\overline{n+1}|} - \frac{1}{{}_nE_x}}$$

↳ Know these GR

18 $\boxed{Var(\bar{a}_{\overline{T_x}|}) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2}}$ & $x:\overline{n}|$ analogy $\quad Var(\ddot{a}_{\overline{K_x}|}) = \frac{{}^2A_x - A_x^2}{d^2}$

$\boxed{{}^2\bar{A}_x = 1 - (2\delta) \cdot {}^2\bar{a}_x}$ $\quad {}^2A_x = 1 - (2d - d^2) \cdot {}^2\ddot{a}_x$

$E[\bar{a}^2_{\overline{T}|}] = \int_0^\infty \bar{a}^2_{\overline{T}|} \cdot {}_tp_x \mu_{x+t}\,dt$

↳ CPM has some sucky math, avoid unless it 'falls' apart

19

Percentile

Recursive: $\boxed{\ddot{a}_x = v \cdot p_x \cdot \ddot{a}_{x+1} + 1}$ $\qquad a_x = v \cdot p_x \cdot a_{x+1} + v \cdot p_x$

Temp) $\ddot{a}_{x:\overline{n|}} = v \cdot p_x \cdot \ddot{a}_{x+1:\overline{n-1|}} + 1$ $\qquad \bar{a}_x = v \cdot p_x \bar{a}_{x+1} + \bar{a}_{x:\overline{1|}}$

Def) ${}_{n|}\ddot{a}_x = v \cdot p_x \cdot {}_{n-1|}a_{x+1}$

20

$\boxed{\ddot{a}_x^{(m)} = \ddot{a}_x - \frac{m-1}{2m}}$ $\longrightarrow$ breaks for big IR

low mortality

$\ddot{a}_x^{(m)} = \alpha(m)\,\ddot{a}_x - \beta(m)$

---

21 Equivalence: APV(prem) = APV(benefits)

$P = \frac{A}{a}$ Tough ones: There is (NO $a^{1}_{x:\overline{n|}}$) So $\bar{P}(\bar{A}^{1}_{x:\overline{n|}}) = \frac{\bar{A}^{1}_{x:\overline{n|}}}{\bar{a}_{x:\overline{n|}}} \rightarrow \frac{1-\bar{A}_{x:\overline{n|}}}{\delta}$

$\bar{P}({}_{n|}\bar{A}_x) = \frac{{}_{n|}\bar{A}_x}{\bar{a}_{x:\overline{n|}}}$ $\qquad \bar{P}({}_{n|}\bar{a}_x) = \frac{{}_{n|}\bar{a}_x}{\bar{a}_{x:\overline{n|}}}$

- Just think about when you're paying ($Pa = A$)

---

22 $\boxed{P_x = \frac{1}{\ddot{a}_x} - d}$ $\qquad \boxed{P_x = \frac{d A_x}{1 - A_x}}$

"Three-Premium Principle"

23

$$\boxed{\begin{aligned} Var({}_0L) &= Var(v^T)\left(1+\frac{P}{\delta}\right)^2 \\ &= \left({}^2\bar{A}_x - \bar{A}_x^2\right)\left(1+\frac{P}{\delta}\right)^2 \end{aligned}}$$

$\Longrightarrow$ GR from ${}_0L = v^T - P\left(\frac{1-v^T}{\delta}\right)$

$$= v^T\left(\frac{1+P}{\delta}\right) - \frac{P}{\delta}$$

$$= Var(v^T)\cdot\left(\frac{1+P}{\delta}\right)^2 + 0$$

(all formulas work for A_x, $A_{x:\overline{n|}}$)

24

25 Special Var case for equiv

$$\boxed{Var({}_0L) \overset{equiv}{=} \frac{{}^2A_x - A_x^2}{(d\ddot{a}_x)^2} \overset{equiv}{=} \frac{{}^2A_x - A_x^2}{(1-A_x)^2}}$$

* Can calc var off moments too

26